Comment on "Thermal Effects on the Casimir Force in the 0.1-5 μm Range" by M. Boström and Bo E. Sernelius (Physical Review Letters 84, pp. 4757-4760 (2000))

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In a recent paper [1] it is shown that simultaneous consideration of the thermal and finite conductivity corrections to the Casimir force between metal plates results in a significant deviation from previous theoretical results [2] and a recent experiment [3]. In [1], it is argued that the TE electromagnetic mode does not contribute as $\omega \to 0$ for a realistic material with dissipation (see Eq. (6) of [1]).

When the zero-frequency limit is taken in [1], it is asserted that

$$\lim_{\omega \to 0} (\gamma_1 - \gamma_0) = 0 \tag{1}$$

where $\gamma_{0,1} = q^2 + \epsilon_{0,1}(i\omega)\omega^2/c^2$, 0 refers to the space between the plates (vacuum, so $\epsilon_0(i\omega) = 1$) and 1 refers to the region within the plates. In [1], the assertion given in Eq. (1) above is based on the idea that, as $\omega \to 0$, $\omega^2\epsilon_1(i\omega) \to 0$ with the assumed form of the dielectric function, Eq. (6) of [1].

In taking the limit in this manner, some important physics is missed. The two evanescent wave numbers, γ_0 and γ_1 describe the damping of the electric field in the space between the plates and within the plates, respectively. It is well-know that for static electric fields that the electric field is perpendicular to a conducting surface (even a poor conductor) and that the field penetration depth into the conductor is infinitesimally small (e.g., the field is cancelled due to charge accumulation on the surface). This implies that $\gamma_1 \to \infty$ when $\omega \to 0$, while γ_0 remains finite.

In taking the zero-frequency limit, the proper result can obtained in a transparent fashion if instead we consider γ_1/γ_0 which should diverge at zero frequency. The point that was missed in [1] is that any non-zero q must be associated with a non-zero ω because the plates are uncharged, and any differential accumulation of charge on the plate surfaces (e.g., due to thermal fluctuations) will be dynamic; q and ω cannot be taken as independent integration variables, but are related in such a way that when $\omega \to 0$ then $q \to 0$.

Using the substitution $q = \omega\sqrt{p^2 - 1}/c$ where $p \geq 1$ (as did Lifshitz [4]; this particular substitution simplifies later integrals), we find that

$$\frac{\gamma_1}{\gamma_0} = \frac{\sqrt{p^2 - 1 + \epsilon_1(i\omega)}}{p} \tag{2}$$

which indeed diverges as $\omega \to 0$, as expected based on the simple physical arguments presented earlier, implying that in this limit

$$G^{TE} = 1 - e^{-2\gamma_0 d} \tag{3}$$

(which is Eq. (2) of [1]). Carrying through this substitution (see [5], pp. 227-228 and pp. 269-271), the previously obtained result is reproduced [2,4,5] for which there is no significant deviation between theory and experiment.

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[4] E.M. Lifshitz, JETP **2**, 73 (1956).

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